Instability of a lattice semifluxon in a current-biased 0-π array of Josephson junctions

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(Received 22 August 2003; published 18 March 2004)

We consider a one-dimensional parallel biased array of small Josephson junctions with a discontinuity point characterized by a phase jump of π in the phase difference. The system is described by a spatially nonautonomous discrete sine-Gordon equation. It is shown that in the infinitely long case there is a semifluxon spontaneously generated attached to the discontinuity point. Comparing the configurations of the semifluxon, we find an energy barrier similar to the Peierls-Nabarro barrier. We calculate numerically the minimum bias current density to overcome this barrier which is a function of the lattice spacing. It is found that the minimum bias current is the critical current for the existence of static lattice semifluxons. For bias current density above the minimum value, the semifluxon changes the polarity and releases 2π fluxons. An analytical approximation to the critical current as a function of the lattice spacing is presented.

DOI: 10.1103/PhysRevB.69.092507 PACS number(s): 74.20.Rp, 74.50.+r, 85.25.Cp, 63.90.+t

The non-*s*-wave superconductivity of high-T*c* superconductors opens the possibility of an intrinsic π phase shift in a Josephson junction, or in other words an effective negative critical current. This possibility is caused by a predominant $d_{x^2-y^2}$ pairing symmetry.[1] A Josephson junction with a discontinuity point characterized by a phase jump of π in the phase difference is called a 0-π junction. Recently, Hilgenkamp *et al.*[2] reported their successful experiments in making $YBa_2Cu_3O_7$-Nb zigzag ramp-type junctions with several discontinuity points.

One important property of a 0-π junction is a spontaneously generated fractional flux quantum or semifluxon that is attached to the discontinuity point.[2] The presence of a semifluxon has been considered before by several authors.[2–7] Initially, such a spontaneous fluxon is used to probe the symmetry of a superconducting gap. It has been conjectured that semifluxons can be utilized in superconducting memory and logic devices since they can change their polarities and "shoot" 2π fluxons under the influence of a certain bias current density.[2,6] The minimum bias current density is called the critical current.[6–8] In experiments this manipulation of semifluxons is done by applying a local field supplied by a currentcoil on a SQUID.[9] Semifluxons can be used also for a more accurate measurement of the Josephson penetration depth.[7]

So far, the previous discussions are mainly focused on semifluxons of long Josephson junctions. Since arrays of 0-π Josephson junctions can be fabricated using the same techniques as described in Refs. 2 and 10 we are led to consider lattice semifluxons of discrete 0-π Josephson junctions. A simple geometry is sketched in Fig. 1(a). One point of difference between an array of short Josephson junctions and a long Josephson junction is that the critical current of the junctions array is higher than that of the continuum case, as will be shown later. In this paper we study numerically and analytically the semifluxon dynamics of a one-dimensional (1D) 0-π array of Josephson junctions for different lattice spacing.

A 1D 0-π array of Josephson junctions is described by a discretized version of a spatially nonautonomous sine-Gordon equation

$$\ddot{\phi}_n-\frac{\phi_{n-1}-2\phi_n+\phi_{n+1}}{a^2}=-\sin[\phi_n+\theta(n)]-\alpha\dot{\phi}_n+\gamma, \quad (1)$$

where $1\leqslant n\leqslant 2N$, $2N$ is the number of junctions/lattices and ϕ_n is the phase difference of the nth junction. In the simulations we take $N=\max\{10/a,50\}$. A time derivative of ψ is denoted by $\dot{\psi}$. Equation (1) is in nondimensionalized form. Parameter α is the positive damping coefficient related to tunneling of electrons across the junctions and $\gamma>0$ is the bias current density. The function $\theta(n)$ is defined as

$$\theta(n)=\begin{cases}0, & 1\leqslant n\leqslant N\\ \pi, & N<n\leqslant 2N,\end{cases} \quad (2)$$

to denote the phase jump of π in the phase difference.

In the limit $a\to 0$ Eq. (1) corresponds to the differential equation for a long 0-π Josephson junction. Thus, for an array of Josephson junctions with small value of a one might expect a dynamical behavior similar to that of the continuum limit. It means we can predict the minimum bias current for a semifluxon to release 2π fluxons in this case. The semifluxon admitted by Eq. (1) for $\gamma=0$ in the limits $a\to 0$ is given by

$$\phi_n^0=\begin{cases}4\arctan\exp[x(n)-x_0], & x(n)<0\\ 4\arctan\exp[x(n)+x_0]-\pi, & x(n)\geqslant 0,\end{cases} \quad (3)$$

where $x(n)=(n-N)a$ and $x_0=\ln(\sqrt{2}+1)$.

The fabrication of the junction as well as the numerics is, of course, limited to finite *N*. One then deals with boundary conditions. A reasonable choice is to take a boundary condition representing the way in which the applied magnetic field *h* enters the system,

$$\frac{\phi_2-\phi_1}{a}=\frac{\phi_{2N}-\phi_{2N-1}}{a}=h=0. \quad (4)$$

The spontaneously generated lattice semifluxon attached to the discontinuity point for two different values of *a* is presented in Fig. 1(b).

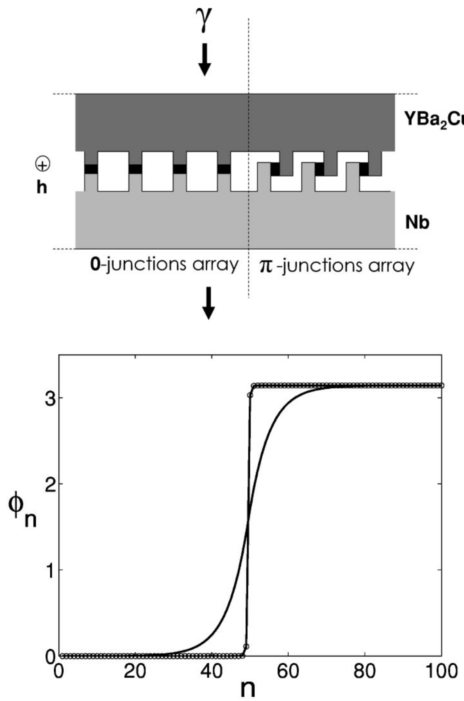

FIG. 1. (a) A sketch of a simple model of the 0-π junction array considered in this report. The sketch is made after Ref. 10. (b) The spontaneously generated lattice semifluxon is plotted as a function of the lattice index. The solid line is the kink for a very weakly discrete array $a=0.2$, i.e., close to Eq. (3), and the other is the kink for a very strongly discrete array $a=5$ (-∘-).

When $\theta(n)$ is constant for all n, Eq. (1) is known also as the Frenkel-Kontorova (FK) model. It has been well-discussed that in the FK model there is a barrier which is called the Peierls-Nabarro (PN) barrier. This PN barrier corresponds to the energy difference of a 2π-kink centered on a lattice site and one centered between two consecutive lattice sites. Because the latter configuration has a minimum energy, one can guess that the spontaneous semifluxon also prefers to sit in the middle between two lattices. The configuration of a lattice semifluxon with low energy is sketched in Fig. 2(a). The other possible configuration [Fig. 2(b)] is unstable, requiring a higher energy. Therefore we observe also an energy barrier similar to the PN barrier.

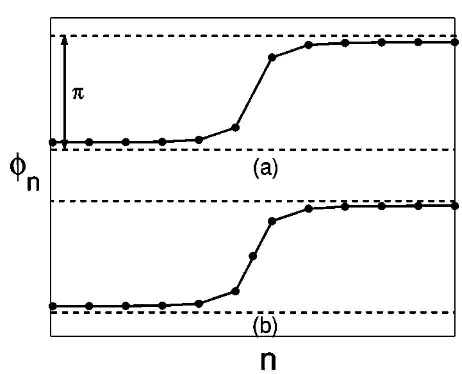

FIG. 2. Stationary configurations of particles in Eq. (1) that correspond to the lattice semifluxon; (a) stable and (b) unstable.

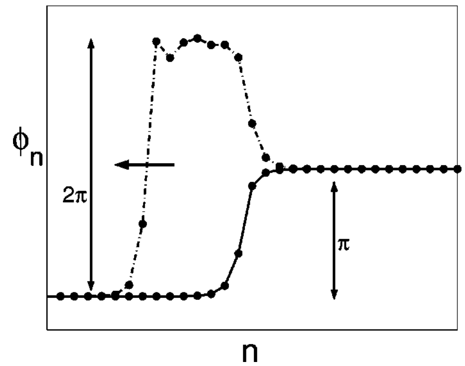

FIG. 3. A sketch of the transition from a lattice π-kink (-∘-) into a lattice π-antikink when we apply a bias current larger than the minimum bias current to overcome the PN-like barrier. The release of an integer antifluxon can be seen as well.

An interesting question is then to determine how large the energy barrier of the semifluxon is for a given value of the lattice spacing. To simplify the calculation, we compute the energy barrier in terms of the applied bias current γ.

We have calculated numerically the minimum bias current to overcome the energy barrier as a function of a. At this value of γ, the lattice configuration is given in Fig. 2(b). This result shows that a lattice semifluxon can move along the junction with maximum displacement half of the lattice spacing. Another interesting result is that for an applied bias current γ larger than this minimum value the static lattice semifluxon does not exist. The semifluxon becomes dynamic and changes its polarity into a lattice π-antikink and release an antifluxon. This means the energy barrier is equal to the critical current for the existence of a static lattice semifluxon γ_c. A sketch of the transition is presented in Fig. 3. As long as γ is larger than γ_c the process repeats itself. One difference between the moving fluxons of the array and the long 0-π Josephson junction is that the moving lattice fluxons emit radiation at the tails because of the discreteness effect. The plot of γ_c as a function of the lattice spacing a is presented in Fig. 4.

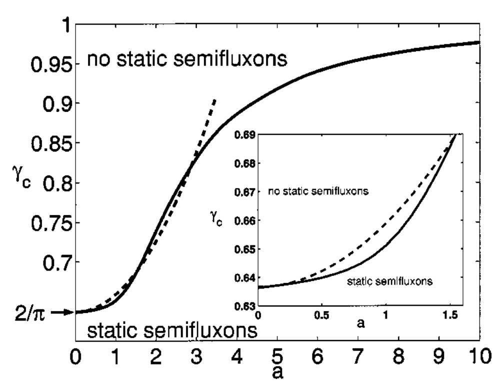

FIG. 4. Plot of the critical bias current density γ_c as a function of the lattice spacing a. This maximum supercurrent is also the energy to overcome the PN-like barrier. The inset magnifies them where it becomes clear that the approximate functions are in a good agreement with the numerical result for small a.

In the case $a\to 0$, γ_c has been calculated as $2/\pi$.[6–8] For $a\ll 1$, the critical bias current is close to this value which means that there is a similarity between a long 0-π Josephson junction and a weakly discrete Josephson transmission line. From $a=1$ to approximately $a=5$ there is a significant increment in the value of the critical current. For $a>5$ the change is not so drastic anymore which means that the semifluxon of Eq. (1) ceases to distinguish the difference of the lattice spacings in a very strongly discrete 0-π Josephson junction. In the limit $a\to\infty$, $\gamma_c\to 1$, as will be shown later. This result is also similar to the PN barrier of a lattice 2π-kink of the ordinary discrete sine-Gordon equation where the energy barrier shows a rapid increase near the value $a=1$. The transition marks the difference between the values of a where the discreteness effects are negligible and values of a where the kink dynamics are dominated by the discreteness.[11]

Next, we will derive an analytical approximation to the critical bias current as a function of a in two different limiting cases, i.e., $a\to 0$ and $a\to\infty$. Since the critical bias current corresponds to the disappearance of the stable static semifluxon, to determine γ_c we only consider the time-independent equation of Eq. (1).

In the limit of $a\to\infty$, the equation is given by

$$\sin[\phi_n+\theta(n)]=\gamma, \tag{5}$$

where the stable semifluxon (mod 2π) is

$$\phi_n=\begin{cases}\arcsin\gamma, & \text{for } n<N\\ \pi+\arcsin\gamma, & \text{for } n\geq N.\end{cases} \tag{6}$$

When $\gamma>1$, there are no solutions to Eq. (5), implying that $\gamma_c\to 1$ when $a\to\infty$.

Analytical calculation of γ_c as a function of a for $a\ll 1$ will be done by considering a continuum approximation of Eq. (1). We will derive a continuum approximation in the region $x<0$ and $x>0$ separately. For simplicity, consider only the discrete sine-Gordon equation for $x<0$

$$\frac{1}{a^2}(\phi_{n-1}-2\phi_n+\phi_{n+1})=\sin(\phi_n)-\gamma. \tag{7}$$

Expanding the left-hand side of the above equation and setting $\phi_n=\phi$, one obtains

$$L_A[\partial_x^2\phi]=\sin\phi-\gamma, \tag{8}$$

where

$$L_A=1+\frac{a^2}{12}\partial_x^2+\frac{a^4}{360}\partial_x^4+\cdots.$$

Considering that ϕ_x is a slowly decaying solution and a is small, one can take the following formal expansion[12]

$$L_A^{-1}=1-\frac{a^2}{12}\partial_x^2+O(a^4).$$

Applying L_A^{-1} to Eq. (8), we obtain a continuum approximation of Eq. (7) [up to $O(a^4)$]

$$\phi_{xx}=\sin\phi-\gamma-\frac{a^2}{12}[\phi_{xx}\cos\phi-\phi_x^2\sin\phi]. \tag{9}$$

Doing the same procedure to the discrete sine-Gordon equation for $x>0$, an approximate equation of Eq. (1) is then given by

$$\phi_{xx}=\sin[\phi+\theta(x)]-\gamma-\frac{a^2}{12}\{\phi_{xx}\cos[\phi+\theta(x)] - \phi_x^2\sin[\phi+\theta(x)]\}, \tag{10}$$

where $\theta(x)$ is defined similar to Eq. (2), i.e.,

$$\theta(x)=\begin{cases}0, & x<0\\ \pi, & x\geq 0.\end{cases}$$

The boundary conditions at the discontinuity point $x=0$ are given by[6–8]

$$\lim_{x\uparrow 0}\phi=\lim_{x\downarrow 0}\phi,\quad \lim_{x\uparrow 0}\phi_x=\lim_{x\downarrow 0}\phi_x.$$

The approximate first integral of Eq. (10) is

$$\frac{1}{2}\phi_x^2=\begin{cases}\cos\phi_- -\cos\phi-\gamma(\phi-\phi_-) \\ \quad -\dfrac{a^2}{12}[\phi_x^2\cos\phi], & x<0\\ \cos\phi-\cos\phi_+ -\gamma(\phi-\phi_+) \\ \quad +\dfrac{a^2}{12}[\phi_x^2\cos\phi], & x\geq 0,\end{cases} \tag{11}$$

$\phi_\pm=\lim_{x\to\pm\infty}\phi$.

Next, it is natural to expand all the quantities as

$$\phi\approx\phi^0+a^2\phi^1$$

and

$$\gamma\approx\gamma^0+a^2\gamma^1$$

from which $\gamma_c\approx\gamma_c^0+a^2\gamma_c^1$. Substituting the expansions to Eqs. (10) and (11) for $O(a^2)$ we obtain the following equations

$$\phi^1_{xx}=\begin{cases}\phi^1\cos\phi^0-\gamma^1-\frac{1}{12}[\phi^0_{xx}\cos\phi^0-\phi^{02}_x\sin\phi^0], & x<0\\ -\phi^1\cos\phi^0-\gamma^1+\frac{1}{12}[\phi^0_{xx}\cos\phi^0-\phi^{02}_x\sin\phi^0], & x\geqslant 0\end{cases}\tag{12}$$

$$\phi^0_x\phi^1_x=\begin{cases}\phi^1\sin\phi^0-\phi^1_-\sin\phi^0_--\gamma^0(\phi^1-\phi^1_-)-\gamma^1(\phi^0-\phi^0_-)-\frac{1}{12}[\phi^{02}_x\cos\phi^0], & x<0\\ \phi^1_+\sin\phi^0_+-\phi^1\sin\phi^0-\gamma^0(\phi^1-\phi^1_+)-\gamma^1(\phi^0-\phi^0_+)+\frac{1}{12}[\phi^{02}_x\cos\phi^0], & x\geqslant 0,\end{cases}\tag{13}$$

where $\phi^i_\pm=\lim_{x\to\pm\infty}\phi^i$. For $O(1)$ the equations are Eqs. (10) and (11) with $\phi=\phi^0$ and $a=0$. It is easy to find out that $\phi^0_-=\arcsin\gamma^0=\phi^0_+-\pi$. We also obtain that $\phi^1_-=\gamma^1/\sqrt{1-\gamma^{02}}=\phi^1_+$.

It is shown in Refs. 6–8 that the critical current corresponds to the condition that

$$\lim_{x\uparrow 0}\phi_{xx}=\lim_{x\downarrow 0}\phi_{xx}\ \text{or}\ \lim_{x\uparrow 0}\phi^i_{xx}=\lim_{x\downarrow 0}\phi^i_{xx},\quad i=0,1,\tag{14}$$

i.e., the appearance of the zero eigenvalue of the semifluxon. For $a=0$ it has been calculated[6–8] that the condition is satisfied when

$$\phi^0(0)=\pi,\qquad \phi^0_x(0)^2=2[\sqrt{1-\gamma^{02}_c}-1+\gamma^0_c\arcsin(\gamma^0_c)]$$

from which the critical bias current of $O(1)$ is derived as $\gamma^0_c=2/\pi$.

Next we will calculate γ^1_c. From Eq. (14), we obtain $\phi^1(0)=-\gamma^0_c/12$. Using the continuity condition that $\lim_{x\uparrow 0}\phi^1_x=\lim_{x\downarrow 0}\phi^1_x$, from Eq. (13) we obtain

$$\gamma^1_c=\frac{\sqrt{1-\dfrac{4}{\pi^2}}\pi-\pi+2\arcsin\left(\dfrac{2}{\pi}\right)}{3\pi^2}\approx 0.02234.\tag{15}$$

It is shown in Fig. 4 that the approximate function has a good agreement with the numerical result.

To conclude, we have presented some numerical results on the lattice array of 0-π Josephson junctions. It is shown that the array has a spontaneously generated semifluxon attached to the discontinuity point. We have observed that the semifluxon can move along the junctions because of the two possible configurations of the semifluxon. We also have computed the energy difference of the two configurations that is similar to the PN barrier as a function of the lattice spacing. It turns out that the energy barrier corresponds to the critical bias current of the existence of the static lattice semifluxon. An approximate function of the critical current has been calculated as well in two limiting cases, i.e., $a\to 0$ and $a\to\infty$. Since the critical bias current increases when the lattice spacing increases, lattice 0-π junctions enlarge the possibility of utilizing the semifluxons in memory cells or logic devices.

H.S. is indebted to H. Hilgenkamp, Darminto, and T.P. Valkering for many stimulating discussions. He thanks also J. R. Kirtley for the information of Ref. 9. This work was supported by the Royal Netherlands Academy of Arts and Sciences (KNAW).

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[1] C.C. Tsuei and J.R. Kirtley, Rev. Mod. Phys. **72**, 969 (2000).

[2] H. Hilgenkamp *et al.*, Nature (London) **422**, 50 (2003).

[3] R.G. Mints *et al.*, Phys. Rev. Lett. **89**, 067004 (2002).

[4] M. Chandran and R.V. Kulkarni, Phys. Rev. B **68**, 104505 (2003).

[5] E. Goldobin, D. Koelle, and R. Kleiner, Phys. Rev. B **67**, 224515 (2003).

[6] A.B. Kuklov, V.S. Boyko, and J. Malinsky, Phys. Rev. B **51**, 11 965 (1995); **55**, 11 878(E) (1997).

[7] T. Kato and M. Imada, J. Phys. Soc. Jpn. **66**, 1445 (1997).

[8] H. Susanto *et al.*, Phys. Rev. B **68**, 104501 (2003).

[9] B.W. Gardner *et al.*, Appl. Phys. Lett. **80**, 1010 (2002).

[10] Darminto and H. Hilgenkamp (private communications).

[11] M. Peyrard and M. Remoissenet, Phys. Rev. B **26**, 2886 (1982).

[12] P. Rosenau, Phys. Rev. B **36**, 5868 (1987).