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Effects of time-periodic linear coupling on two-component Bose–Einstein condensates in two dimensions

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Abstract

We examine two-component Gross–Pitaevskii equations with nonlinear and linear couplings, assuming self-attraction in one species and self-repulsion in the other, while the nonlinear inter-species coupling is also repulsive. For initial states with the condensate placed in the self-attractive component, a sufficiently strong linear coupling switches the collapse into decay (in the free space). Setting the linear-coupling coefficient to be time-periodic (alternating between positive and negative values, with zero mean value) can make localized states quasi-stable for the parameter ranges considered herein, but they slowly decay. The 2D states can then be completely stabilized by a weak trapping potential. In the case of the high-frequency modulation of the coupling constant, averaged equations are derived, which demonstrate good agreement with numerical solutions of the full equations.

Keywords: Nonlinear Schrödinger equations; Multiple components; Linear coupling

1. Introduction

Studies of matter-wave patterns in Bose–Einstein condensates (BECs), and especially solitons, have drawn a great deal of interest from experimentalists [1] and theorists [2,3] alike. In effectively one-dimensional (1D) traps ("cigar-shaped" ones), solitons have been created in condensates of ^{7}Li and ^{85}Rb atoms [4]. The attractive interaction between atoms, necessary for this, can be provided even in condensates with naturally repulsive interactions by switching the interaction type, with the help of the Feshbach resonance (FR) through an external magnetic field [5]. In the repulsive condensates, solitons of the gap type, supported by the periodic potential induced by optical lattices (OLs), have been predicted [6] and then demonstrated experimentally in the 1D setting (in the condensate of ^{87}Rb atoms) [7].

The creation of matter-wave solitons in 2D and 3D settings has not been reported yet, the most fundamental difficulty being the possibility of collapse in the same geometry [8], which makes the multidimensional solitons unstable. Various ways to stabilize 2D and 3D solitons against the collapse have been elaborated theoretically. One of them is the use of OLs, whose dimension may be equal to that of the underlying space [9], or smaller than it by 1 [10]. Another method relies on periodic time modulation of the effective strength of the nonlinear interaction, through the FR induced by an ac magnetic field. For the first time, a similar possibility was explored in a model of the propagation of (2 + 1)D spatial optical solitons (cylindrical beams) shone through a layered medium, which is built of periodically alternating layers with positive and negative values of the Kerr coefficient [11] (recently, the stabilization of spatial solitons of this type was demonstrated experimentally in an optical medium composed of

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alternating nonlinear and linear layers [12]). Directly in terms of the 2D BEC, the possibility to stabilize solitons by means of this *FR-management* technique, with the nonlinearity coefficient periodically alternating in time between positive and negative values, was predicted and analyzed in Refs. [13,14] (the same approach does not provide for the stabilization of 3D solitons, but it can do so in a combination with the quasi-1D OL potential [15]; the method does not stabilize 2D vortex solitons either [11]). In all these cases, it was found that the stabilization of the 2D solitons requires the time-average value of the nonlinearity coefficient to be different from zero, corresponding to the self-attraction (or self-focusing, in the optical medium). Still, it may happen that this stabilization mechanism generates an extremely long-lived transient dynamical regime, rather than a truly stable one: super-long simulations (on the time scale several orders of magnitude larger than the duration relevant to any experiment) reveal the beginning of what may be a very slow decay of the soliton through emission of radiation [16].

In the 1D geometry, the same FR-management technique gives rise to specific dynamical states of the condensate trapped in the static parabolic potential, such as breathers oscillating between Thomas–Fermi and quasi-soliton configurations, and stable two-soliton bound states [17]. Other states, predicted as a result of the interplay between the low-frequency modulation of the nonlinearity coefficient and the OL potential (in the 1D and 2D geometries alike) are *alternate solitons*, which periodically switch between gap-soliton and ordinary solitonic shapes [18]. It is relevant to mention that the FR-management technique belongs to the class of *soliton-management* methods, which were developed in nonlinear optics and then applied to BEC [19].

The effective stabilization of 2D solitons by means of the FR-management technique suggests another possibility: as is known, atoms of a given element (in particular, ^{7}Li) may exist in different intrinsic states, some of which interact repulsively between themselves, while others—attractively, the interaction between atoms belonging to the different states being repulsive in any case [4]; then, an external electromagnetic wave, resonantly coupling such two states, may give rise to oscillations between numbers of atoms in the self-attractive and self-repulsive states, and thus, possibly, help to stabilize 2D solitons against the collapse [20]. The objective of the present Letter is to report the first results of the theoretical analysis of the 2D model of this type, which may help to direct further studies of the topic. When the linear-coupling coefficient is subject to periodic modulation in time (in fact, periodic jumps between positive and negative values, with zero mean value), we conclude that the 2D soliton can be made quite long-lived, in the free space. If, in addition, a weak external trapping potential is added, the waveform features complete stabilization.

The above-mentioned resonant electromagnetic wave induces linear coupling between the wave functions of the two species (atomic states) in the respective system of coupled Gross–Pitaevskii equations (GPEs). A similar linear coupling (accounting for the interconversion between two states) was investigated in a number of settings (usually, assuming that the coupled species represent two different spin states of the same atom, while the electromagnetic wave is the spin-flipping one). Effects and patterns predicted in the framework of models with this type of the linear coupling include Josephson-like oscillations [21], domain walls [22], “breathe-together” oscillation modes [23], non-topological vortices [24], and a shift of the miscibility transition in binary Bose–Bose [25] and Fermi–Fermi [26] gases. However, to the best of our knowledge, effects of time-modulated (“managed”) linear coupling have not been considered before, in this context.

The rest of the Letter is organized as follows. In Section 2, we introduce the model. In Section 3, its simplified version is derived, by means of the averaging method (assuming high-frequency time modulation of the linear-coupling coefficient). Numerical results, which support the basic inferences outlined above, are presented in Section 4, and the Letter is concluded by Section 5.

2. The model

The model is based on a pair of coupled 2D GPEs for mean-field wave functions of the two species, u and v. The general form of these equations is well known [27]. In the standard scaled form, it is

$$\begin{aligned} iu_t &= -(1/2)\nabla^2 u + \left(g_1|u|^2 + |v|^2\right)u + \epsilon^{-1}\kappa(t/\epsilon)v + (1/2)\Omega^2 r^2 u, \\ iv_t &= -(1/2)\nabla^2 v + \left(g_2|v|^2 + |u|^2\right)v + \epsilon^{-1}\kappa(t/\epsilon)u + (1/2)\Omega^2 r^2 v, \end{aligned} \tag{1}$$

where kinetic-energy operator $(1/2)\nabla^2$ acts on spatial coordinates x and y, Ω^2 is the strength of the isotropic trapping potential ($r^2 \equiv x^2 + y^2$), the coefficient accounting for the nonlinear repulsion between the species is normalized to be 1, and, in accordance with what said above, we assume the attraction between the atoms in the first species, and repulsion in the second, i.e., $g_1 < 0$ and $g_2 > 0$. Further, the form of the time-dependent linear-coupling coefficient, $\epsilon^{-1}\kappa(t/\epsilon)$, assumes a possibility of the high-frequency modulation corresponding to small ϵ (see below), while the function $\kappa(\tau)$ is chosen in the following form ($\tau \equiv t/\epsilon$),

$$\kappa(\tau) = \begin{cases} M, & 0 < \mathrm{mod}(\tau, T) < T/2, \\ -M, & T/2 < \mathrm{mod}(\tau, T) < T, \end{cases} \tag{2}$$

with period T.

Throughout this Letter, we fix the nonlinearity coefficients in Eqs. (1) to be $g_1 = -1$ and $g_2 = 1.03$. The proximity of the intra-species repulsive coefficient (g_2) to the one accounting for the inter-species repulsion (recall it was normalized to be 1) corresponds to the usual physical situation for ^{87}Rb, see, e.g., Ref. [28]. As for the intra-species attraction coefficient g_1 (which in ^{87}Rb could be produced by a Feshbach resonance), the choice of $g_1 = -1$ is a natural one, as we expect that the self-attraction and self-repulsion in the species coupled by the linear interconversion tend to be in balance. In fact, additional simulations have demonstrated that other values of $g_1 < 0$ and $g_2 > 0$ lead to essentially the same results as demonstrated below. As for the strength of the trapping potential, it will be either 0 or $\Omega^2 = 0.02$.

It is relevant to mention a somewhat similar model of two parallel cigar-shaped (quasi-1D) traps, coupled by tunneling, in the case when the application of the FR (in dc magnetic field) to one trap makes the signs of the nonlinearity coefficients opposite in the two respective GPEs. In that case, the nonlinear interaction between the two wave functions is absent, while the coefficient of the linear interaction, accounting for the tunnel coupling between the parallel traps, is constant. Solitons in the respective 1D model were studied in detail [29], and the 2D generalization, corresponding to two parallel pancake-shaped traps was introduced too [30] (the latter does not give rise to stable solitons). We note in passing that in such a "double well" setting, time dependent linear coupling (induced by a time-dependent trap) was considered also in [31].

3. Averaged equations

Before proceeding to numerical results, it is relevant to consider possibilities of an analytical approximation. We tried to employ the variational approximation based on the Gaussian ansatz, but we were not able to produce predictions complying with the numerical results. More reasonable analytical results can be obtained in the limit of the high-frequency modulation (as said above, this corresponds to small ϵ in Eq. (1)), which allows us to apply the averaging method. The latter method proved to be quite relevant in the studies of the stabilization of 2D solitons by means of the FR-management technique [13], and was also applied to other BEC models involving rapid temporal modulation of the trapping potential [32] or nonlinearity coefficient [33].

The first step is to eliminate the linear-coupling terms in Eq. (1) by means of the transformation similar to that employed in Ref. [34],

$$\begin{pmatrix}\phi\\ \psi\end{pmatrix} \equiv \begin{pmatrix}\cos(\kappa_{-1}) & i\sin(\kappa_{-1})\\ i\sin(\kappa_{-1}) & \cos(\kappa_{-1})\end{pmatrix}\begin{pmatrix}u\\ v\end{pmatrix}, \tag{3}$$

$$\kappa_{-1}(\tau) = \int_0^\tau \kappa(\tau')\,d\tau' - \int_0^1\int_0^\tau \kappa(\tau')\,d\tau'\,d\tau. \tag{4}$$

Notice that in Eq. (4), for piecewise constant $\kappa(\tau)$, one integration may be performed trivially $[\kappa_{-1}(\tau) = \int_0^\tau \kappa(\tau')\,d\tau' - \int_0^1 \kappa(\tau')\tau'\,d\tau']$. However, we keep the expression in its general form, as the periodic dependence may not necessarily be piecewise constant. The substitution of the above change of variables in Eq. (1) yields a system of transformed equations,

$$\begin{aligned} i\phi_t = {} & -(1/2)\nabla^2\phi + (1/2)\Omega^2 r^2\phi + \big(C_1|\phi|^2 + C_2|\psi|^2\big)\phi - C_3\psi^2\phi^* \\ & + C_3\cos(4\kappa_{-1})\big[\psi^2\phi^* + \big(|\phi|^2 - 2|\psi|^2\big)\phi\big] + 2C_4\cos(2\kappa_{-1})|\phi|^2\phi \\ & + iC_3\sin(4\kappa_{-1})\big[\phi^2\psi^* + \big(-2|\phi|^2 + |\psi|^2\big)\psi\big] + iC_4\sin(2\kappa_{-1})\big[\phi^2\psi^* - \big(2|\phi|^2 + |\psi|^2\big)\psi\big], \end{aligned} \tag{5}$$

$$\begin{aligned} i\psi_t = {} & -(1/2)\nabla^2\psi + (1/2)\Omega^2 r^2\psi + \big(C_2|\phi|^2 + C_1|\psi|^2\big)\psi - C_3\phi^2\psi^* \\ & + C_3\cos(4\kappa_{-1})\big[\phi^2\psi^* + \big(|\psi|^2 - 2|\phi|^2\big)\psi\big] - 2C_4\cos(2\kappa_{-1})|\psi|^2\psi \\ & + iC_3\sin(4\kappa_{-1})\big[\psi^2\phi^* + \big(-2|\psi|^2 + |\phi|^2\big)\phi\big] - iC_4\sin(2\kappa_{-1})\big[\psi^2\phi^* - \big(2|\psi|^2 + |\phi|^2\big)\phi\big], \end{aligned} \tag{6}$$

where $C_1 = (3g_1 + 3g_2 + 2)/8$, $C_2 = (g_1 + g_2 + 2)/4$, $C_3 = (g_1 + g_2 - 2)/8$, $C_4 = (g_1 - g_2)/4$. Next, making use of the assumed smallness of ϵ, we expand the wave functions $\phi = \sum_{k=0}^\infty \epsilon^k \Phi_k(\tau, t_0, t_1, \ldots)$ and $\psi = \sum_{k=0}^\infty \epsilon^k \Phi_k(\tau, t_0, t_1, \ldots)$ (recall $\tau \equiv t/\epsilon$), using multiple-scale variables $t_n \equiv \epsilon^n t$, such that $d/dt = \epsilon^{-1}\partial_\tau + \partial_{t_0} + \epsilon\partial_{t_1} + \cdots$.

At order ϵ^{-1}, expansion of Eqs. (5) and (6) yields $(\Phi_0)_\tau = (\Psi_0)_\tau = 0$, which means that Φ_0 and Ψ_0 do not depend on τ. At the next order, a closed-form system of equations is derived for Φ_0 and Ψ_0,

$$i(\Phi_0)_t = -(1/2)\nabla^2\Phi_0 + (1/2)\Omega^2 r^2\Phi_0 + \big[(C_1 + \sigma_1 C_3 + 2\sigma_2 C_4)|\Phi_0|^2 + (C_2 - 2\sigma_1 C_3)|\Psi_0|^2\big]\Phi_0 - C_3(1 - \sigma_1)\Psi_0^2\Phi_0^*, \tag{7}$$

$$i(\Psi_0)_t = -(1/2)\nabla^2\Psi_0 + (1/2)\Omega^2 r^2\Psi_0 + \big[(C_2 - 2\sigma_1 C_3)|\Phi_0|^2 + (C_1 + \sigma_1 C_3 - 2\sigma_2 C_4)|\Psi_0|^2\big]\Psi_0 - C_3(1 - \sigma_1)\Phi_0^2\Psi_0^*, \tag{8}$$

where $\sigma_n = \int_0^1 \cos(2n\kappa_{-1}(\tau))\,d\tau$ with $n = 1, 2$. Expressions for corrections Φ_1 and Ψ_1 can also be found from the expansion at order ϵ^0:

$$\begin{aligned} i\Phi_1 &= C_3\{\cos(4\kappa_{-1}) - \sigma_1\}_{-1}\big[\Phi_0^2\Phi_0^* + \big(|\Phi_0|^2 - 2\big|\Psi^0\big|^2\big)\Phi_0\big] \\ &\quad + 2\{\cos(2\kappa_{-1}) - \sigma_2\}_{-1} C_4|\Phi_0|^2\Phi_0 + iC_3\{\sin(4\kappa_{-1})\}_{-1}\big[\Phi_0^2\Psi_0^* + \big(-2|\Phi_0|^2 + |\Psi_0|^2\big)\Phi_0\big] \\ &\quad + iC_4\{\sin(2\kappa_{-1})\}_{-1}\big[\Phi_0^2\Psi_0^* - \big(2|\Phi_0|^2 + |\Psi_0|^2\big)\Psi_0\big], \end{aligned} \tag{9}$$

$$\begin{aligned} i\Psi_1 &= C_3\{\cos(4\kappa_{-1}) - \sigma_1\}_{-1}\big[\Phi_0^2\Psi_0^* + \big(|\Psi_0|^2 - 2|\Phi_0|^2\big)\Psi_0\big] \\ &\quad + 2\{\cos(2\kappa_{-1}) - \sigma_2\}_{-1} C_4|\Psi_0|^2\Psi_0 + iC_3\{\sin(4\kappa_{-1})\}_{-1}\big[\Psi_0^2\Phi_0^* + \big(-2|\Psi_0|^2 + |\Phi_0|^2\big)\Phi_0\big] \\ &\quad - iC_4\{\sin(2\kappa_{-1})\}_{-1}\big[\Psi_0^2\Phi^{0*} - \big(2|\Psi_0|^2 + |\Phi_0|^2\big)\Phi_0\big], \end{aligned} \tag{10}$$

where functions $\{\cdots\}_{-1}$ are defined in Eq. (4).

4. Numerical analysis

4.1. Basic results

The simulations of the underlying system of Eqs. (1), presented below, were performed with all atoms put, at the initial moment ($t = 0$), in the self-attractive component (the one which is prone to collapse). A typical example of the ensuing dynamical behavior is represented by numerical solutions corresponding to initial conditions

$$u(x, y) = 2.25\exp\big(-r^2/4\big), \qquad v(x, y) = 0. \tag{11}$$

The choice of this initial state places the system beyond the collapse threshold, i.e., it features the collapse in the absence of the linear coupling. Simulations performed with the constant linear coupling in Eqs. (1), $\kappa/\epsilon = \text{const}$, clearly demonstrate that localized (soliton-like) states *cannot* be stabilized in this way for the combination of nonlinear coefficients: there is a critical value, $(\kappa/\epsilon)_{\text{cr}} \approx 3.9$ (for initial conditions (11)), of the coupling constant, below which the two-component state still collapses, while at $\kappa/\epsilon > (\kappa/\epsilon)_{\text{cr}}$ it suffers dispersing (decay), with some oscillations (in fact, these are Rabi oscillations between the two atomic states, with time period $\pi\epsilon/\kappa$ [25,34,35], induced by the linear coupling). In Fig. 1, this inference is illustrated by examples of the collapse and decay at $\kappa/\epsilon \equiv 3$ and $\kappa/\epsilon \equiv 5$, respectively (top left and right panels), in the free space ($\Omega = 0$). This result can be understood, given the fact that, if the Rabi oscillations are slow (at small κ), they do not prevent the collapse at the stage when a bigger part of the condensate is represented by the self-attractive species; on the other hand, if the oscillations are fast, the averaged nonlinearity is

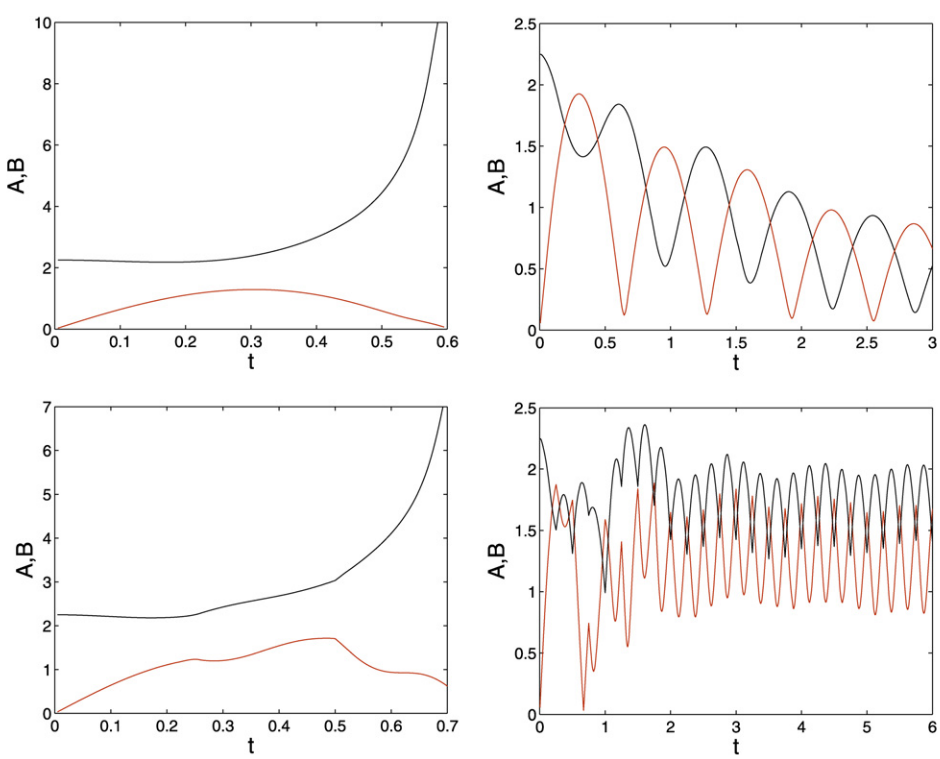

Fig. 1. The evolution of initial state (11) produced by simulations of underlying equations (1). Shown are amplitudes of both components, $\{A(t), B(t)\}$ with a constant (time-independent) linear-coupling coefficient (the top row), $\kappa/\epsilon = 3$ and 5, in the left and right panels, respectively. The bottom panels show the same, but for κ modulated in time as per Eq. (2), with temporal period $\epsilon T = 1/2$.

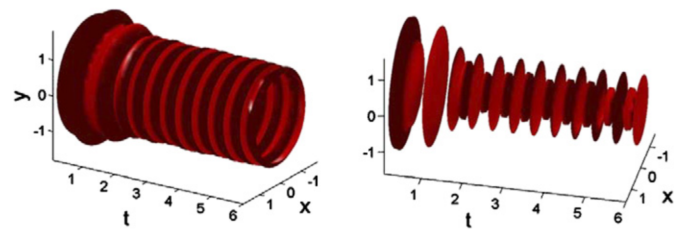

Fig. 2. Space–time evolution of the density isosurfaces with $|u(x,y,t)| = 1$ and $|v(x,y,t)| = 1$ for the case of the bottom right panel of Fig. 1.

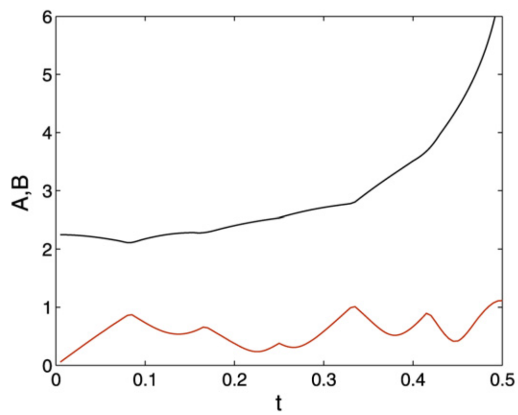

Fig. 3. The same as in the bottom right panel of Fig. 1 (with modulation amplitude in Eq. (2) $|\kappa|/\epsilon = 5$), but for a short modulation period, $\epsilon T = 1/6$.

self-repulsive (taking into regard the inter-species repulsion, see Eqs. (7) and (8)), which naturally leads to the eventual dispersion of the condensate.

The next step is the application of the "management" technique. To this end, we simulated Eqs. (1) with the linear coupling periodically modulated in time as per Eq. (2). As the lower part of Fig. 1 shows, the temporal modulation does not essentially affect the collapse of the localized state in the subcritical regime (for $|\kappa|/\epsilon = 3$, see the left bottom panel). However, it is quite interesting that the "management" provides *long-term stabilization* of the quasi-soliton beyond the critical value of the coupling constant—in particular, at $|\kappa|/\epsilon = 5$, as shown in the left bottom panel of Fig. 1; see also the space–time evolution of density isosurfaces in Fig. 2. Very long simulations demonstrate that the seemingly stabilized localized state actually suffers slow decay, which can be seen clearly at $t > 100$. It may be that this slow decay of the stabilized 2D soliton is similar to the above-mentioned decay occurring in the case of the stabilization provided by the FR management [16]. Detailed analysis of the slow delay is a challenging problem that is outside of the scope of the present Letter.

Besides the fact that a time-varying linear coupling can decrease the decay rate of a 2D soliton, it is also worthy to note that, making the management period smaller, it is possible for the quasi-stabilization of the 2D localized states to be replaced by an *induced collapse* (i.e., the collapse taking place in the case when the localized state decays in the absence of the management). An example of that is shown in Fig. 3, for the same situation as in the bottom right panel of Fig. 1, but with the modulation period three times as short, $\epsilon T = 1/6$. This can be intuitively understood in the following way: the Rabi oscillations are used in the present setting to "convert" matter from one component to the other. Contrary to what is the case in Fig. 1, the periodic change of κ in Fig. 3 reverses the "flow of matter" before the attractive component has lost enough mass to actually disperse. As a result, the sign reversal leads the attractive component to regain "mass" and eventually to collapse.

To prevent a collapse from happening for a small oscillation period, one needs a relatively large amplitude of oscillations for the linear coupling. Then, one will obtain decaying states which is in a way more favorable as full stabilization in this case is possible if a weak trapping potential is included in Eq. (1) ($\Omega \neq 0$).

In the following subsection, we will consider this particular case, i.e., the case of linear coupling with relatively small oscillation period and large amplitude of oscillations, and compare the original governing equation (1) and the averaged equations (7)–(8).

4.2. *Comparison with the averaged equations, and complete stabilization*

It is quite interesting to compare direct simulations of Eqs. (1) with results provided by the averaged equations (7) and (8), taking into regard relations (3) and (4) between the variables for which the two systems are written. In particular, the form of the nonlinear terms in Eqs. (7) and (8) suggests that, in the free space (with $\Omega = 0$), these equations cannot give rise to a stable 2D soliton for the parameter ranges considered herein, which will indeed be demonstrated below.

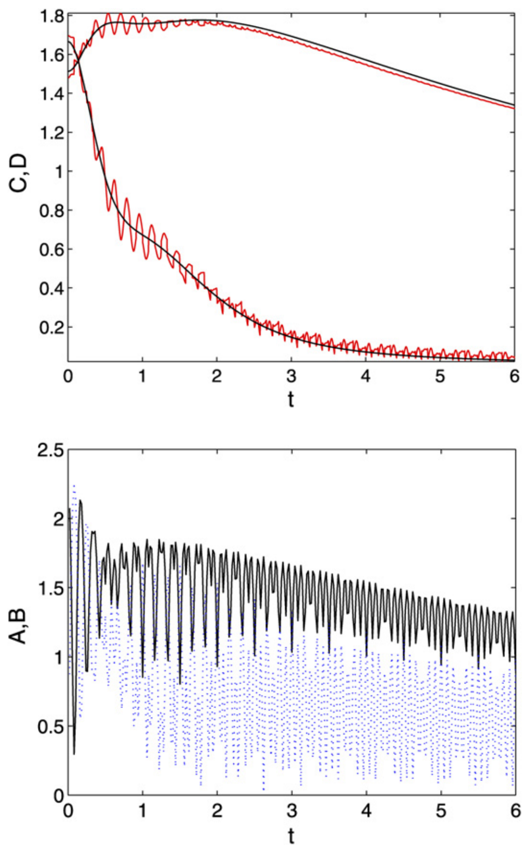

Fig. 4. Top panel: Comparison of the evolution of the amplitudes of the two components of the localized state, as found from simulations of the averaged equations (7) and (8) (smooth curves) and full equations (1) (wiggling curves). Both solutions are shown in terms of variables (ϕ, ψ). Bottom panel: the same solutions to full equations (1), but shown in terms of the original variables (u, v), to which (ϕ, ψ) are related as per Eq. (3). Here, the linear coupling is given by expression (2) with $\epsilon = 0.1$ and $\epsilon T = 1/6$.

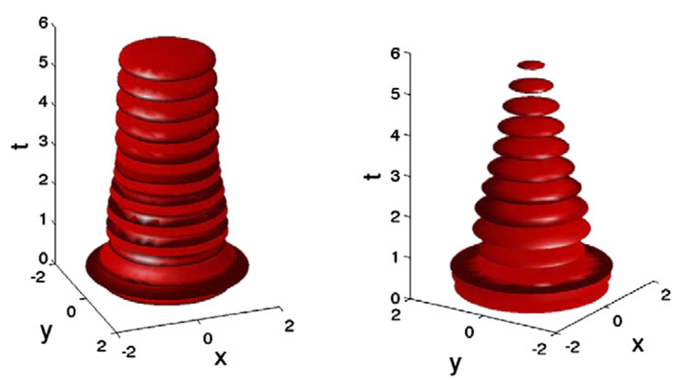

Fig. 5. Density isosurfaces of the two components of the localized 2D state from Fig. 4, defined as surfaces at which $|u(x, y, t)| = 1$ and $|v(x, y, t)| = 1$.

To comply with the condition of the applicability of the averaging method, we present results of the comparison for $\epsilon = 0.1$, see Eqs. (1) (without the potential trap, $\Omega = 0$). A typical example of the comparison is displayed in Fig. 4. We notice that the averaged equations yield quite an accurate description of the behavior of the full system.

The spatiotemporal evolution of the same 2D state whose amplitudes are shown, as functions of time, in Fig. 4, is displayed by means of density isocontours (level surfaces) of components u and v in Fig. 5. This rendition makes it obvious how the 2D state decays (the self-repulsive component disperses quickly, and the self-attractive one follows it, but more slowly).

Systematic simulations performed at many values of the parameters have demonstrated that, without the external trapping potential, the stabilization of the 2D localized states against the collapse by means of the time-periodic linear coupling is quite possible, but the full stabilization (against the slow decay) is not. On the other hand, the inclusion of the trap makes it easy to demonstrate permanent stabilization (which is not surprising, as it is quite plausible that the confining potential should prevent decay of the soliton into dispersive waves). A generic example of the complete stabilization is presented in Fig. 6, which shows that even the state

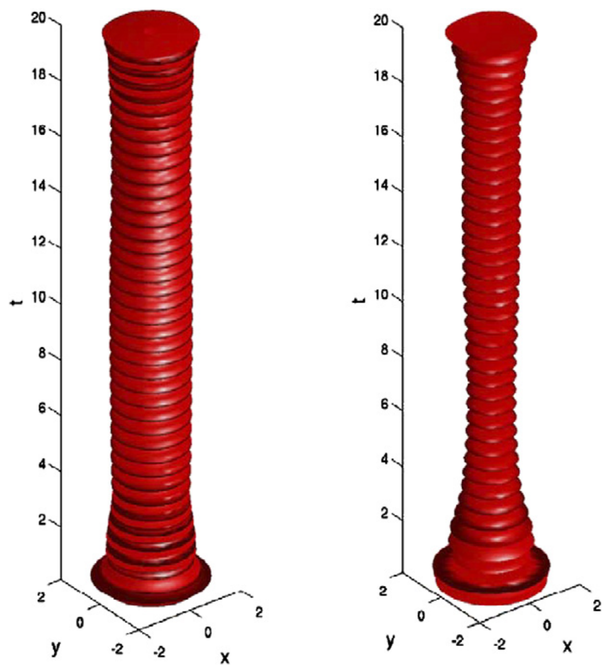

Fig. 6. The same as in Fig. 5, but in the presence of a weak trapping potential in Eq. (1), with $\Omega^2 = 0.02$.

from Figs. 4 and 5, which was clearly unstable against the decay in the free space, is readily stabilized by a weak trap. Oscillations of the resulting structure, observed in the figure, continue indefinitely long without initiating any instability (while, in the absence of the time-periodic linear coupling, the same state quickly collapses).

5. Conclusions

In this Letter, we have investigated the possibility of stabilization of 2D soliton-like states, composed of self-attractive and self-repulsive components which are coupled by the linear interconversion and feature mutual nonlinear repulsion. It was demonstrated that the linear coupling with the constant coefficient can switch the collapse of the self-attractive component into its decay (parallel to the decay of the self-repulsive counterpart). The periodic time modulation of the linear-coupling coefficient (which puts the model in the broad class of the "soliton management") allowed us to find quasi-stable soliton-like states, which avoid collapse, but eventually suffer slow decay. Permanent stabilization of such states is provided by the inclusion of a weak trapping potential. For the case of the high-frequency time modulation, averaged equations have been derived, which demonstrate good accuracy in comparison with direct simulations of the underlying system. It would be interesting to examine similar questions and generalize the considerations presented herein in a fully three-dimensional setting, both in the presence and in the absence of a magnetic trap. Such studies are in progress and will be reported in future publications.

Note added in proof

A similar model was recently studied in preprint "Stabilization of a Bose–Einstein droplet by hyperfine Rabi oscillations", by H. Saito, R.G. Hulet, and M. Ueda, arXiv: 0707.4530.

The parameter region considered in that work was different from the one dealt with in the present paper (it corresponded to strong linear coupling, without the time modulation). It was concluded that a narrow stability domain for solitons, between regimes of expansion and collapse, could be found in the respective parameter space. An analytical approximation developed in the preprint effectively reduces the system, by means of a linear transformation of the two wave functions, to the model considered in Refs. [13] and [14], where the stabilization is possible indeed, although possibly as a very long-lived transient [16]. Thus, a more extensive exploration of the various dynamical regimes in the entire parameter space of the model would be particularly relevant.

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