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Fluxons interactions in a Josephson junction with a phase-shift

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ABSTRACT

We consider a long Josephson junction with a discontinuity point characterized by a gauge phase-shift. The system is described by a modified sine-Gordon equation. We study, in particular, the interactions between a fluxon and a fractional fluxon. A perturbation theory is developed in the small phase-shift limit to understand the characteristics of the interaction. Finally, numerical computations of the threshold bias current and the threshold velocity for a fluxon running over a fractional fluxon are presented.

1. Introduction

The idea of having a phase-shift in the gauge phase of a Josephson junction was first proposed by Bulaevskii et al. [1,2]. It was proposed that the presence of magnetic impurities may create a π phase-shift to the Josephson phase. Recent experimental results have confirmed this conjecture [3]. Present technological advances can also impose a π-phase-shift in a long Josephson junction using, e.g., superconductors with unconventional pairing symmetry [4], Superconductor–Ferromagnet–Superconductor (SFS) π-junctions [5], or Superconductor–Normal metal–Superconductor (SNS) junctions in which the microscopic current-carrying electronic states in the weak conduction channels are controlled [6]. It is also important to note that recently Goldobin et al. [7] have reported a successful experiment making a long Josephson junction with an arbitrary κ phase-shift using a pair of current injectors. All these findings have promising applications in information storage and information processing [8]. It is therefore interesting also from a physical point of view to study the effects of such an inhomogeneity on a moving fluxon, which inspires the investigations in the present work.

A Josephson junction with a phase-shift of κ is described in nondimensionalized form by [9]

$$\phi_{xx} - \phi_{tt} - \sin\big(\phi - \kappa H(x)\big) = \alpha\phi_t + \gamma, \tag{1}$$

where ϕ is the phase difference of the order parameter in the superconductors, $\alpha \geqslant 0$ is the dissipation constant related to the passage of a normal current across the junction, and γ is the applied bias current. Due to the symmetry $(\phi, \kappa, \gamma) \to (-\phi, -\kappa, -\gamma)$, the value of the applied force γ can be limited to $\gamma \geqslant 0$. The Heaviside function $H(x)$ represents the phase-shift with the discontinuity at $x = 0$. It is then trivial to notice that κ can be scaled to $0 \leqslant |\kappa| \leqslant 2\pi$.

Taking the transformation $u(x,t) = \phi(x,t) - \kappa H(x)$, the quantity u then satisfies [9]

$$u_{xx} - u_{tt} - \sin(u) = \alpha u_t + \gamma - \kappa\delta_x(x). \tag{2}$$

Interestingly, this equation was first proposed by Aslamazov and Gurevich [10] to describe fluxons dynamics under the presence of Abrikosov vortices in one of the superconductors near the junction (see also [11] and references therein for more theoretical works and [12] for experimental reports).

The dynamics of fluxons in a Josephson system with phase-shifts have been considered before by several authors. For the particular case of interactions between a fluxon and a discontinuity point, one can see [13–16]. Interactions between a fluxon and two discontinuity points have been discussed as well in [17–19]. The dynamics of a fluxon in a Josephson junction with periodic and aperiodic infinite discontinuities have also been studied in [20–22].

In this work, we revisit the problem of fluxon dynamics in a Josephson system with one discontinuity point. This case is still of interest especially since most of the previous works either did not include the presence of the so-called fractional fluxon [25,26], which is a non-zero ground-state, or assumed smallness to the phase-shift κ. The inclusion of the ground state necessarily complicates the problem. Therefore, in this study we rely heavily on numerical computations.

The present report is organized as follows. First, we discuss the so-called fractional fluxon which is the background of the dis-

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continuous system. We then describe and derive the perturbation theory developed in, e.g., [15,27,23,24] to study the dynamics of a single fluxon in the presence of one discontinuity (2) valid for the case of $|\kappa| \ll 1$. We also calculate the threshold values of the velocity and the bias current for the fluxon escape and trapping. Next, we solve Eq. (1) numerically and compare the result of the preceding section. We conclude the work afterwards.

2. Pinned fractional fluxons

An undriven ($\gamma = 0$) Josephson junction with a discontinuity point (2) has the following stable non-constant background [25,26]

$$u_f(x,\kappa) = \begin{cases} 4\arctan\{e^x \tan\frac{\kappa}{8}\}, & x < 0, \\ -4\arctan\{e^{-x}\tan\frac{\kappa}{8}\}, & x > 0, \end{cases} \tag{3}$$

pinned to the discontinuity point. When $|\kappa| \ll 1$, expression (3) can be approximated by [15]

$$u_f \approx -\frac{\kappa}{2}\,\mathrm{sgn}(x)\exp(-|x|). \tag{4}$$

One can notice that ϕ satisfying Eq. (1) with a phase-shift of κ also satisfies the same equation with a phase-shift of $(\kappa \pm 2\pi)$. Therefore, for one value of phase-shift, there are actually two types of fractional fluxons to Eq. (2), namely $u_f(x,\kappa)$ and $-\mathrm{sgn}(\kappa)u_f(x,\kappa - \mathrm{sgn}(\kappa)2\pi)$. The ground state of the system is then a fractional fluxon with the smallest topological charge, i.e. $|\lim_{x\to\infty} u_f(x) - \lim_{x\to-\infty} u_f(x)|$. To avoid confusion, in the following we will assume that a phase-shift of κ corresponds to a background, i.e. a pinned fractional fluxon, of topological charge κ, $u_f(x,\kappa)$.

3. Damped, driven junctions

Here, we will consider analytically and perturbatively the dynamics of a fluxon in a damped, driven junction with a phase-shift of κ, i.e. $\alpha, \gamma \neq 0$. Kivshar and Chubykalo [15] argue that the dynamics of a fluxon has to be considered with respect to the ground state u_f (4) such that to the leading orders, the governing equation (2) becomes

$$\tilde{u}_{xx} - \tilde{u}_{tt} - \sin(\tilde{u}) = \alpha\tilde{u}_t + \gamma - 2u_f\sin^2(\tilde{u}/2), \tag{5}$$

where $\tilde{u} = u - u_f$. Yet, we will show in this report that an approximation derived using the adiabatic approximation theory [15,27] from Eq. (2), i.e. the background u_f is neglected, still gives a reasonably good approximation to the numerics.

In the absence of perturbations, i.e. $\alpha = \gamma = \kappa = 0$, Eq. (2) supports an exact solitonic solution representing a fluxon in a Josephson junction

$$u_k = 4\arctan\exp\left[\frac{x - \xi(t)}{\sqrt{1-v^2}}\right], \tag{6}$$

where v and $\xi(t) = vt$ are the kink's velocity and the center-of-mass coordinate. In a homogeneous, damped, driven Josephson junction, i.e. Eq. (2) with $\kappa = 0$, the equilibrium velocity v_0 is determined by the balance between the driving force γ and the dissipation α [28,29]:

$$v_0 = \frac{\pi\gamma}{4\alpha\sqrt{1+(\frac{\pi\gamma}{4\alpha})^2}}. \tag{7}$$

In the "nonrelativistic" case, i.e. $0 < v^2 \ll 1$ or $0 < \gamma \ll \alpha$, the fluxon (6) can be approximated by

$$u_k \approx 4\arctan\exp\left[x - \xi(t)\right], \tag{8}$$

such that (7) yields

$$v_0 \approx \pi\gamma/(4\alpha). \tag{9}$$

The momentum of the soliton u_k (8) is defined as [33]

$$P = -\int_{-\infty}^{\infty} u_{kx}u_{kt}\,dx = 8\dot{\xi}(t), \tag{10}$$

which is conserved in the absence of perturbations.

Due to the presence of the perturbations, the momentum changes. Adiabatically, its perturbation-induced evolution is given by

$$\frac{dP}{dt} = -\int_{-\infty}^{\infty}(u_{kx}u_{kt})_t\,dx = \int_{-\infty}^{\infty} u_{kx}(\alpha u_{kt} + \gamma - \kappa\delta_x)\,dx, \tag{11}$$

where we have used the assumption that the soliton tends to a constant in the limit at $x \to \pm\infty$.

Evaluating the integrals yields

$$8\ddot{\xi} = -8\alpha\dot{\xi} + 2\pi\gamma + 2\kappa\tanh\xi\,\mathrm{sech}\,\xi. \tag{12}$$

By writing Eq. (12) as $8\,d^2\xi/dt^2 = -dU/d\xi$, one finally obtains that the dynamics of a fluxon in Eq. (2) can be viewed as the motion of a particle in a potential

$$U = -2\pi\gamma\xi + \underbrace{2\kappa\,\mathrm{sech}\,\xi}_{U_0(\xi)}, \tag{13}$$

under the action of the friction force $F_{\mathrm{fr}} = -8\alpha\dot{\xi}$. The plot of U is presented in Fig. 1. From this, one can see that when the incoming fluxon has the same polarity as the fractional fluxon (that is, when $\kappa > 0$) the fluxon can be repelled by the inhomogeneity. On the other hand, if the fluxons have opposite polarities ($\kappa < 0$), they tend to attract each other [10,15].

A fluxon situated near the local minimum of the potential can be captured by the impurity. Once it is captured, it can be released again by increasing the bias current [13]. The minimum value of bias current for the release of a fluxon is called the critical current γ_{cr}, which has been calculated as [13]

$$\gamma_{\mathrm{cr}} = \pm\frac{2}{2\pi+\kappa}\sin\frac{\kappa}{2}. \tag{14}$$

The 'plus–minus' sign refers to the sign of κ. For the case of $\kappa < 0$, γ_{cr} is obtained by viewing a 2π fluxon trapped by a κ fluxon as a $(2\pi + \kappa)$ fractional fluxon.

If the bias current decreases the fluxon can be captured again by the impurity at a threshold value γ_{thr}. In the following, we will approximate this threshold value from the adiabatic approximate equation (12). An approximation of γ_{thr} can be obtained by assuming that at $\gamma = \gamma_{\mathrm{thr}}$ the velocity v of the incoming fluxon moving in the potential U becomes $v = 0$ at the turning point $\xi = \xi^\star$ indicated in Fig. 1.

Multiplying Eq. (12) with $\dot{\xi}$, integrating it over time t, and using the proper boundary conditions, one will obtain

$$4\dot{\xi}^2\Big|_{-\infty}^{\xi^\star} = -8\alpha\int_{-\infty}^{\xi^\star}\dot{\xi}\,d\xi + 2\pi\gamma\xi\Big|_{-\infty}^{\xi^\star} - U_0\Big|_{-\infty}^{\xi^\star},$$
$$\Leftrightarrow\ -4v_0^2 = -E_{\mathrm{diss}}(\xi^\star) + 2\pi\gamma\xi^\star - U_0(\xi^\star), \tag{15}$$

where we have substituted our assumption that $\dot{\xi} = 0$ at $\xi = \xi^\star$. When $\kappa > 0$ and $\kappa < 0$, $\xi^\star \approx -\pi\gamma/\kappa$ and $\xi^\star \approx \ln(-2\kappa/(\pi\gamma))$, respectively. By assuming that close to the defect $8\ddot{\xi} \approx -dU_0/d\xi$, i.e.

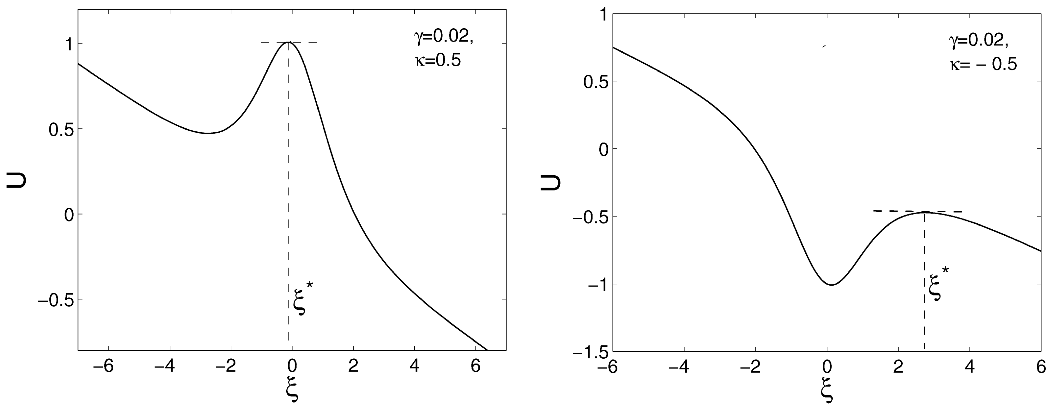

Fig. 1. A sketch of the potential U for the two case of $\kappa > 0$ and $\kappa < 0$. The position of $\xi^\star$ is also depicted in both sketches.

$4\dot{\xi}^2 \approx 4v_0^2 - U_0$ [24], $E_{\text{diss}}(\xi) \approx 8\alpha \int \sqrt{v_0^2 - U_0/4}\, d\xi \approx 8\alpha \int [v_0 - U_0/(8v_0)]\, d\xi = 8\alpha v_0 \xi - 2\kappa\alpha/v_0 \arctan\sinh\xi$. Using Eq. (9), (15) will then determine the threshold bias current γ_{thr} as a function of κ and α.

Using the assumption that $\xi^\star \to 0$ and $\xi^\star \to -\infty$ for the case of $\kappa > 0$ and $\kappa < 0$, Eq. (15) can be evaluated analytically as

$$\gamma_{\text{thr}} = \sqrt{8\alpha^2\kappa/\pi^2}, \qquad \gamma_{\text{thr}} = \sqrt[3]{-16\alpha^4\kappa/\pi^2}, \tag{16}$$

respectively.

4. Undamped, undriven junctions

Even though the case of undamped and undriven junctions, i.e. $\alpha = \gamma = 0$, is a bit unphysical, it can exhibit non-trivial and exotic dynamics, such as fluxon reflections by an attractive impurity [30]. If in the case of damped driven junctions the interactions between a fluxon and a fractional fluxon are characterized by the presence of the threshold bias current γ_{thr} (as a function of the dissipation constant α), in the undamped undriven case, they are characterized by the threshold velocity v_{thr}. Technically, obtaining v_{thr} for a fluxon passing an attractive potential is more involved and delicate in this case [31]. The adiabatic approximation discussed in the previous section does not immediately work [32]. However, Kivshar and Malomed (see p. 840 in Ref. [33]) have proposed a method for the perturbation analysis of Eq. (5) that may be applicable. This method uses the complete integrability of the unperturbed sine-Gordon equation $u_{xx} - u_{tt} = \sin(u)$ and applies the inverse scattering transform. Kivshar and Chubykalo [15] have used this method to study the effect of the bias current γ on the trapping and release of a fluxon governed by Eq. (5).

The critical threshold for the case of $\kappa > 0$ in the undriven undamped system can be simply derived from Eq. (15) with $\xi^* = 0$, from which one will obtain [10]

$$v_{\text{thr}} = \sqrt{\kappa/2}. \tag{17}$$

As for the case of $\kappa < 0$, we do not present any analytical approximation here because of its complex calculations, which will be reported elsewhere.

5. Numerical results

To study the kink scattering by a phase-shift discontinuity, we use a conservative numerical scheme, i.e. the fourth order Runge–Kutta method, to discretize Eq. (1). The simulations are carried out in the spatial interval $(-30, 30)$ with discrete step sizes $\Delta x = 2\Delta t = 0.02$. The initial conditions are taken as a kink centered at $\xi(0) = -15$ incoming with a velocity v toward a fractional fluxon with topological charge κ, $u_f(x, \kappa)$.

In Fig. 2, we show numerical simulations of the typical interactions between an incoming fluxon and a κ fractional fluxon with $|\kappa| \ll 2\pi$ for the case of undamped, undriven junctions. One can see that when $\kappa > 0$ there are two distinct qualitative behaviors, namely the incoming fluxon may be repelled by or pass through the inhomogeneity. Similarly, when $\kappa < 0$ the interaction may result in the incoming fluxon being trapped by or passing through the inhomogeneity. The minimum value of the incoming velocity for a fluxon to pass the inhomogeneity is the threshold velocity v_{thr} discussed in the previous section. Even though we do not depict the simulations for damped, driven junctions, the dynamics are similar and the threshold bias current γ_{thr} corresponds to the same feature as v_{thr}.

In the top left and right panels of Fig. 3, we plot γ_{thr} numerically obtained from solving Eq. (1) for $\alpha = 0.05$ for the case of $\kappa > 0$ and $\kappa < 0$, respectively. In both panels, we also depict our approximation presented as solid red lines, obtained by numerically solving Eq. (15). One can notice that the analytical calculations approximate the numerical results well. Besides Eq. (15), we plot as well the approximation (16) in dashed lines for both cases. The two approximations of γ_{thr} obtained from (15) and (16) in the top left panel may appear to lie on top of each other because for this case the difference is only of the order of 10^{-3}.

As for the case of undamped undriven junctions, we plot the numerically obtained threshold velocity v_{thr} in the bottom left and right panels of Fig. 3 as a function of κ for both $\kappa > 0$ and $\kappa < 0$, respectively. We also plot in the bottom left panel our approximation (17) from which one can also observe a good agreement between the numerics and the approximation in the region of relatively small κ.

6. Conclusion

We have studied the kink dynamics in a Josephson system with a κ phase-shift. The scattering has been explained as well in the framework of perturbation theory for solitons. The behavior is characterized by an effective potential that depends on the fluxon's polarity. The bias current and the velocity threshold for a fluxon passing the discontinuity point have been calculated numerically and analytically. Our considerations are mainly focused on the case of interactions between a fluxon and a fractional fluxon of topological charge κ with $|\kappa| \ll 2\pi$. The case of $|\kappa| \sim 2\pi$ is out of the scope of the present Letter since other phenomena, such as the creation of breathers, can occur and significantly change the qualitative behavior of the solution. In such a situation v_{thr} and γ_{thr} need to be redefined. Yet, as one can view a κ fractional fluxon with $|\kappa| \sim 2\pi$ as an integer fluxon trapped by a $(\kappa - \text{sgn}(\kappa)2\pi)$ phase-shift, this case can then be viewed as an interaction of two integer fluxons in a Josephson junction with $(\kappa - \text{sgn}(\kappa)2\pi)$ phase-

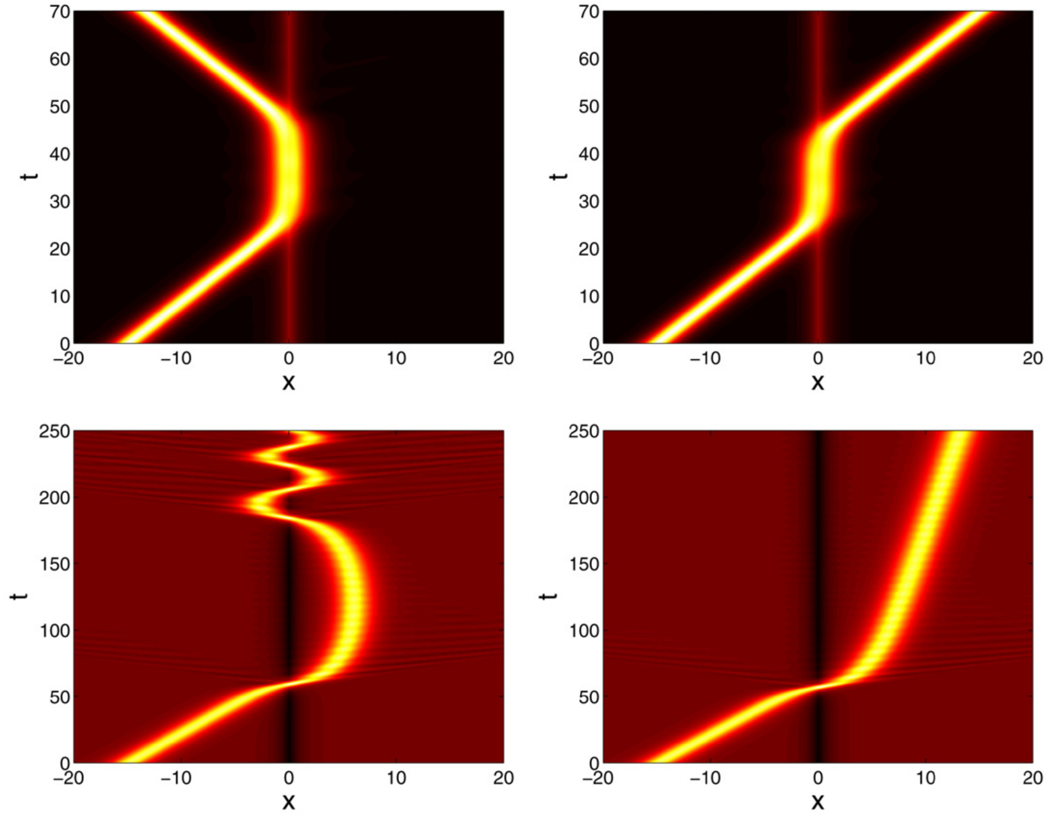

Fig. 2. (Color online.) The dynamics of an incoming fluxon in an undamped, undriven system with $\kappa = 1$ (top panels) and $\kappa = -1$ (bottom panels). Left panels depict the dynamics where the fluxon's incoming velocity is less than the threshold velocity v_{thr}. Clockwise from top left to bottom left panel, the velocity is $v = 0.5702, 0.5703, 0.23, 0.22$.

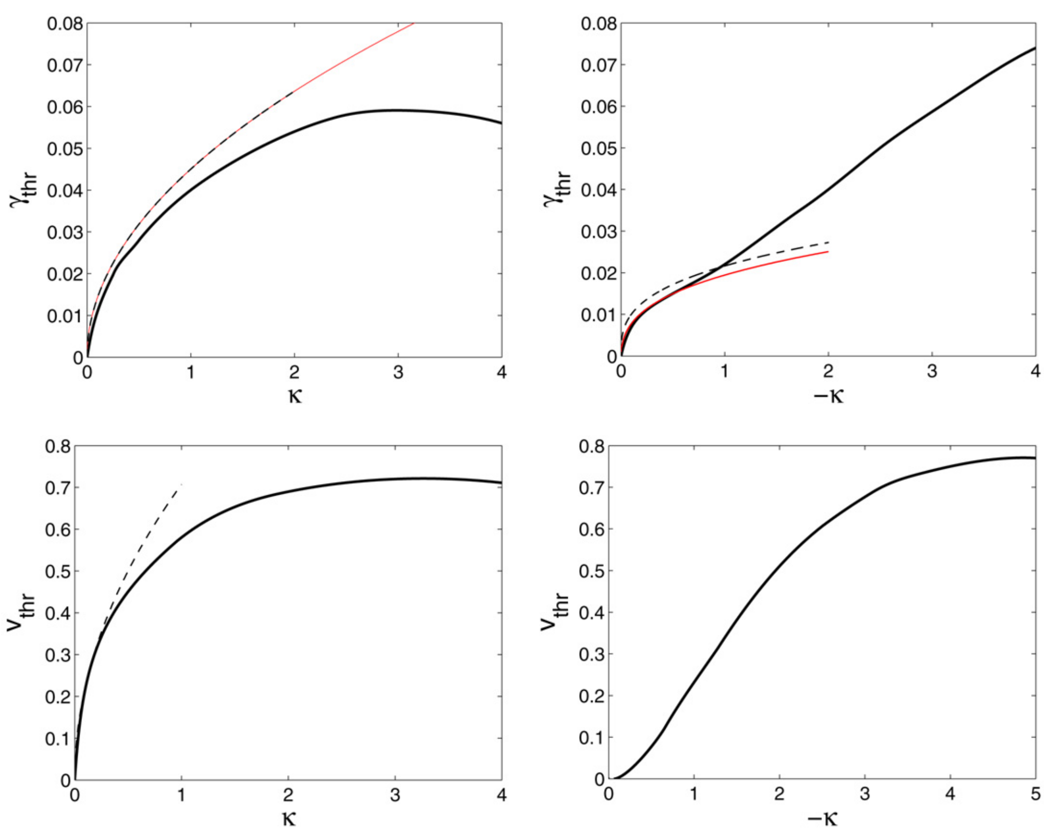

Fig. 3. (Color online.) (Top panels) Numerically obtained threshold bias current γ_{thr} as a function κ for $\alpha = 0.05$. (Bottom panels) Numerically obtained threshold velocity v_{thr} as a function κ in the undamped, undriven case. Solid black lines are numerical results obtained from solving Eq. (1). Solid red lines are obtained from solving (15). Dashed lines are from (16) and (17).

shift. An analytical study is again possible in this limiting case following [16]. This problem is currently under study and will be reported in a future publication.

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